CHAPTER 12

Analyses to Back-Calculate Strengths

When a slope fails by sliding it can provide a useful source of information on the conditions in the slope at the time of the failure as well as an opportunity to validate stability analysis methods. Because the slope has failed, the factor of safety is considered to be unity (1.0) at the time of failure. Using this knowledge and an appropriate method of analysis it is possible to develop a model of the slope at the time that it failed. The model consists of the unit weights and shear strength properties of the soil, groundwater, and pore water pressure conditions and the method of analysis, including failure mechanisms. Such a model can help in understanding the failure better and be used as a basis for analysis of remedial measures. The process of determining the conditions and establishing a suitable model of the slope from a failure is termed *back-analysis* or *back-calculation*.

BACK-CALCULATING AVERAGE SHEAR STRENGTH

The simplest back-analysis is one where an average shear strength is calculated from the known slope geometry and soil unit weights. This is accomplished by assuming a friction angle of zero and calculating a value of cohesion that will produce a factor of safety of 1. This practice of calculating an average strength expressed as a cohesion can, however, lead to erroneous representations of shear strength and potentially unfavorable consequences (Cooper, 1984). For example, consider the natural slope shown in Figure 12.1 and suppose that the slope has failed. We can begin by assuming a value of cohesion and calculating a factor of safety. If we assume a cohesion of 500 psf, the calculated factor of safety is 0.59. The developed cohesion, c_d, can then be calculated as

$$c_d = \frac{c}{F} = \frac{500}{0.59} = 850 \text{ psf} \qquad (12.1)$$

The cohesion developed is the cohesion required for a factor of safety of 1.0. Thus, the back-calculated shear strength is 850 psf. Now, suppose that one remedial measure being considered is to decrease the height of the slope to 30 ft (Figure 12.2). If the slope height is reduced to 30 ft and the cohesion is 850 psf, the new factor of safety is 1.31. Because the shear strength has been calculated from an actual slide, much of the uncertainty normally associated with the measurement of shear strength is eliminated. Thus, a factor of safety of 1.31 may be more than adequate, and based on this analysis we might choose to reduce the slope height to 30 ft as the repair measure.

In the foregoing case we were able to back-calculate an average shear strength expressed as a *cohesion*, c, with $\phi = 0$. Little more can be done if all that we know about the slope is that it failed. However, often there is more information that can be used to obtain a better estimate of the shear strength and other conditions in the slope at the time of failure. Suppose that the slope described above failed many years after the slope was formed. If this is the case, we would analyze the stability using drained shear strengths and effective stresses; we would not consider the friction angle to be zero unless the slope had failed soon after construction. Let's suppose further that from experience with clays like the clay in this slope, we know that the friction angle is about 22° and that there is a small cohesion, c'. Finally, let's suppose that we have found from observations that a piezometric line such as the one shown in Figure 12.3 approximates the seepage conditions in the slope at the time of failure. We can then back-calculate a value for the effective cohesion (c') that will produce a factor of safety of 1.0. The proce-

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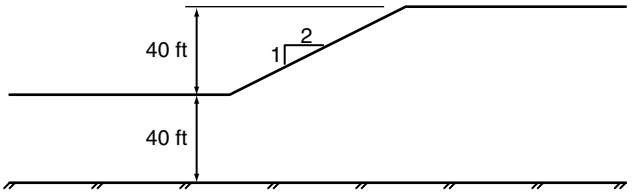


Figure 12.1 Homogeneous natural slope that has failed.

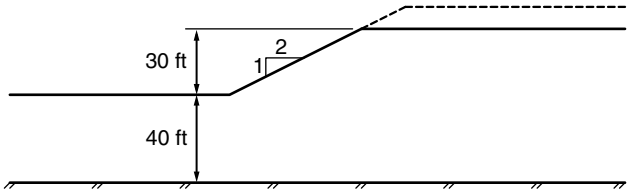


Figure 12.2 Homogeneous slope with reduced height.

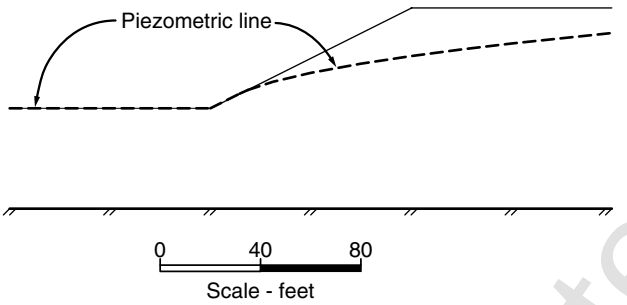


Figure 12.3 Piezometric line for homogeneous slope.

dure for back-calculating the cohesion in this case is slightly different from what was done above. Several values of cohesion need to be assumed. With a friction angle of 22° and the piezometric line shown in Figure 12.3, the factor of safety is calculated for each assumed value of cohesion. The results of such calculations are summarized in Figure 12.4. It can be seen that a cohesion of approximately 155 psf produces a factor of safety of 1.0. Using the shear strength parameters ($c' = 155$ psf, $\phi' = 22°$) determined by back-analysis, we can again calculate the stability of the slope with the height reduced from 40 ft to 30 ft. The factor of safety with the height reduced to 30 ft is 1.04. This factor of safety (1.04) is substantially less than the factor of safety (1.31) determined for the slope when the shear strength was back-calculated as a cohesion with $\phi = 0$.

Next, suppose that for the slope described above, another alternative remedial measure is to lower the water level to the elevation of the toe of the slope. If

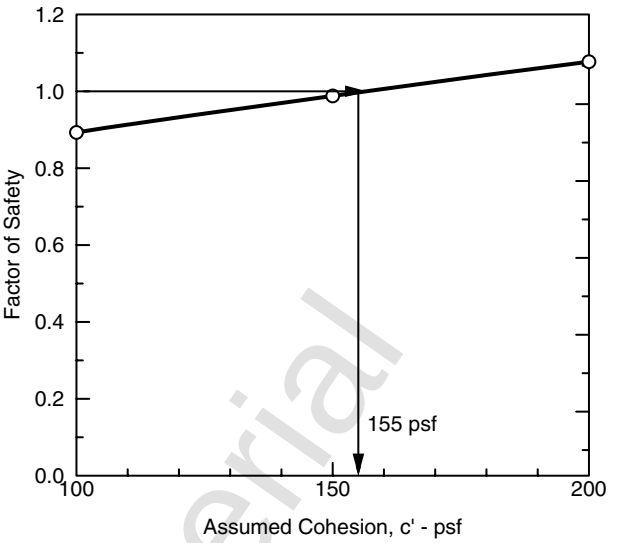


Figure 12.4 Variation in factor of safety with the assumed value for the cohesion (c') for simple homogeneous slope and foundation. $\phi' = 22°$.

we apply the first set of shear strengths that were back-calculated ($c = 850$ psf, $\phi = 0$), we will conclude that lowering the water level has no effect on the factor of safety because the friction angle is zero, and thus the shear strength does not depend on either the total or the effective normal stress. However, if we use the effective stress shear strength parameters ($c' = 155$ psf, $\phi' = 22°$) that were determined by the second back-analysis, the factor of safety is increased to 1.38, which would indicate that lowering the groundwater level would be an acceptable remedial measure.

The results of the back-analyses and the analyses of remedial alternatives described above are summarized in Table 12.1. It can be seen that very different conclusions would be reached regarding the effectiveness of remedial measures, depending on how the shear strength is characterized and what information is used

Table 12.1 Summary of Back-Analyses and Analyses of Remedial Measures for Homogeneous Slope

Shear strength parameters from back-analysis	Factor of safety for remedial measure	
	Decrease slope height to 30 ft	Lower water level to toe of slope
$c = 850$ psf, $\phi = 0$	1.31	1.00
$c' = 155$ psf, $\phi' = 22°$	1.04	1.38

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for back-analysis. For the natural slope that failed a number of years after formation, back-calculation of an average shear strength expressed as cohesion led to an overestimate of the effectiveness of reducing the slope height and an underestimate of the effectiveness of lowering the water level.

By using back-analysis, it is only possible to back-calculate a single shear strength parameter. In the first case summarized in Table 12.1, ϕ was assumed to be zero and an average shear strength, expressed as a cohesion, was back-calculated. In the second case knowledge that the friction angle was approximately 22° and the approximate location of a piezometric line were used to back-calculate an effective cohesion, c'. In both cases, cohesion was back-calculated while the friction angle (ϕ, ϕ') was either assumed or known from other information. It would also be possible to assume that the cohesion (c, c') was zero and to back-calculate a friction angle; however, only one unknown shear strength parameter can be calculated using back-analysis.

Recapitulation

- Only one strength parameter (c, c' or ϕ, ϕ') can be calculated by back-analysis.
- Back-calculation of an average shear strength expressed as a cohesion, c ($\phi = 0$) can produce misleading results when a slope has failed under long-term drained conditions.

BACK-CALCULATING SHEAR STRENGTH PARAMETERS BASED ON SLIP SURFACE GEOMETRY

Although for any given slope there are an infinite number of pairs of values for cohesion (c, c') and friction angle (ϕ, ϕ') that will produce a factor of safety of 1, each such pair of values will also produce a different location for the critical slip surface. This is illustrated for a simple slope in Figure 12.5. Three sets of shear strength parameters and corresponding critical circles are shown. Each set of shear strength parameters produces a factor of safety of 1, but the critical slip surface is different. For a simple homogeneous slope such as the one shown in Figure 12.5, the depth of the slip surface is related to the dimensionless parameter, $\lambda_{c\phi}$, defined as

$$\lambda_{c\phi} = \frac{\gamma H \tan \phi}{c} \tag{12.2}$$

where H is the slope height and c and ϕ represent the appropriate total stress or effective stress, shear strength parameters. Values of $\lambda_{c\phi}$ are shown along with the shear strength parameters in Figure 12.5. As $\lambda_{c\phi}$ increases, the depth of the slip surface decreases. When $\lambda_{c\phi}$ is zero, the slip surface is deep, and when $\lambda_{c\phi}$ is infinite (c, $c' = 0$), the slip surface is shallow—essentially a shallow infinite slope failure. Because each pair of shear strength parameters (c–ϕ or c'–ϕ') corresponds to a unique slip surface, the location of the slip surface, along with the knowledge that the slope has failed (i.e., $F = 1$), can be used to back-calculate values for two shear strength parameters (c–ϕ or c'–ϕ').

To illustrate how the location of the slip surface can be used to back-calculate both cohesion and friction, consider the slope illustrated in Figure 12.6. This is a highway embankment constructed in Houston, Texas, of highly plastic clay, known locally as Beaumont Clay. A slide developed in the embankment approximately 17 years after the embankment was built. The estimated location of the slip surface is shown in Figure 12.6. Because the failure occurred many years after construction, drained shear strengths were assumed and slope stability analyses were performed to calculate shear strength parameters in terms of effective stresses. The pore water pressure was assumed to be zero for these particular analyses. The following steps were performed to back-calculate the shear strength parameters and slip surface location:

1. Several pairs of values of cohesion and friction angle (c' and ϕ') were assumed. The pairs of values were chosen such that they represented a range in the dimensionless parameter $\lambda_{c\phi}$, but the values did not necessarily produce a factor of safety of 1.
2. The critical circles and corresponding minimum factors of safety were calculated for each pair of values of the strength parameters.
3. Values of the developed shear strength parameters (c'_d and ϕ'_d) were calculated for each pair of strength parameters from the following equations using the assumed cohesion and friction angle and the computed factor of safety:

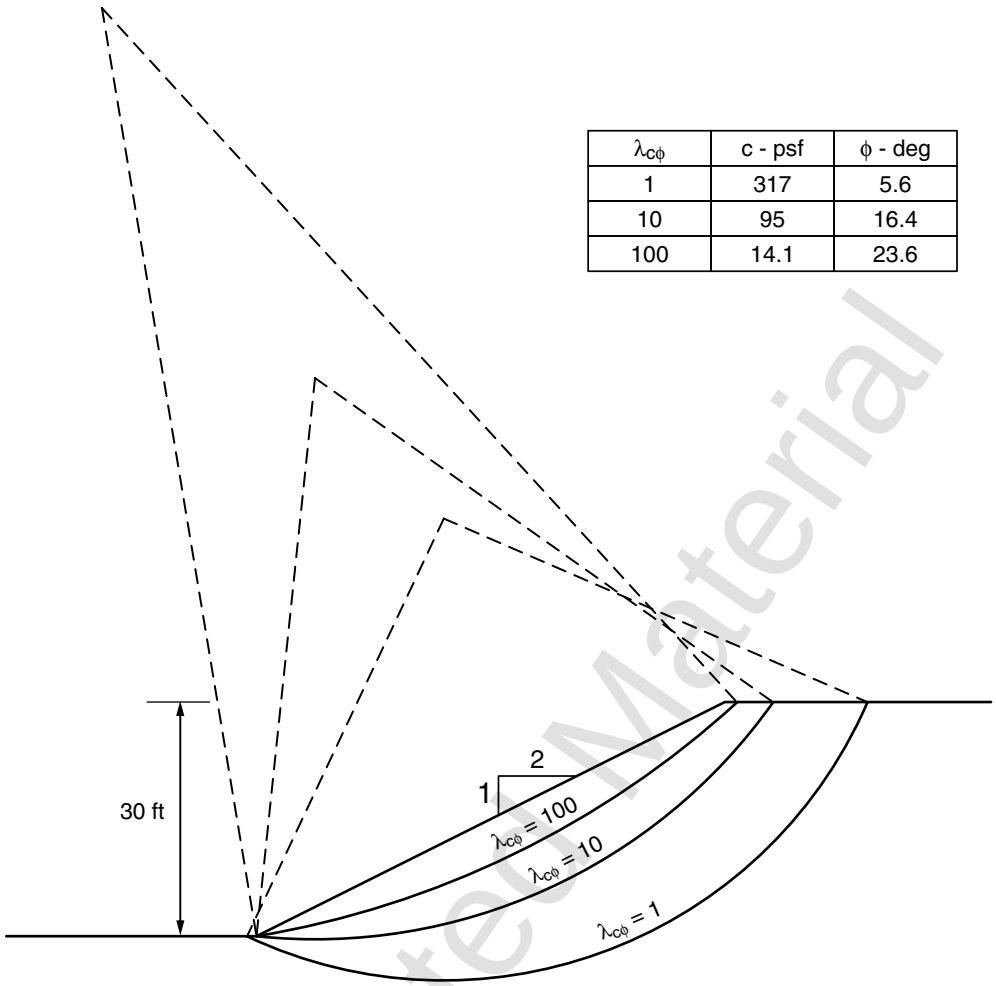
$$c'_d = \frac{c'}{F} \tag{12.3}$$

$$\phi'_d = \arctan \frac{\tan \phi'}{F} \tag{12.4}$$

The developed cohesion and friction angle represent back-calculated values required to produce a factor of safety of 1.

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$\lambda_{c\phi}$	c - psf	ϕ - deg
1	317	5.6
10	95	16.4
100	14.1	23.6

Figure 12.5 Critical circles for three different sets of shear strength parameters giving a factor of safety of 1.

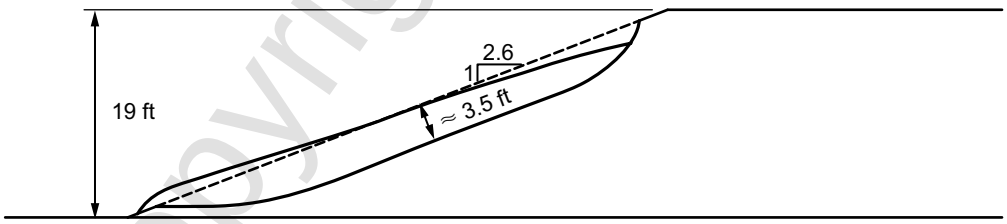


Figure 12.6 Slide in compacted high-PI clay fill.

4. The depth of the critical slip surface for each pair of values of strength parameters was calculated.
5. The back-calculated cohesion and friction angle from step 3 were plotted versus the depth of the slip surface, calculated in step 4 (Figure 12.7).
6. The cohesion and friction angle corresponding to the observed slide depth (3.5) ft were determined from the plotted results.

These steps showed that a cohesion of 5 psf and a friction angle of 19.5° produce a factor of safety of 1 with a slide depth of 3.5 ft. These values seem reasonable for the effective stress shear strength parameters for a highly plastic clay.

Calculations like the ones described above can be simplified by the use of dimensionless stability charts that allow the cohesion and friction angle to be back-

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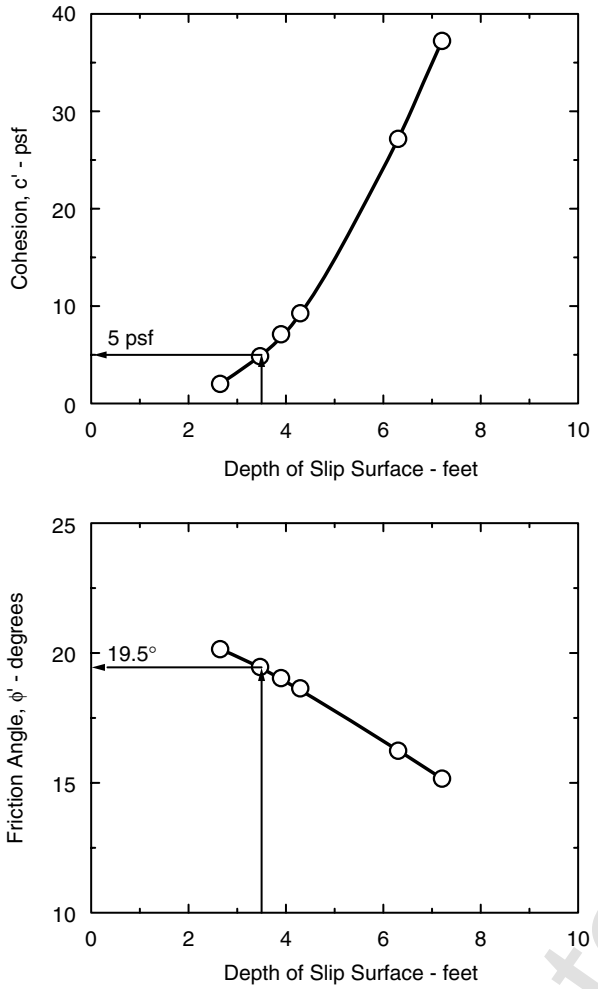


Figure 12.7 Variation in values for cohesion (c') and friction angle (ϕ') that produce a factor of safety of 1 with the depth of the slip surface.

calculated directly. Such charts are based on dimensionless parameters similar to those described for the stability charts used to compute factors of safety, which are described in the Appendix. Abrams and Wright (1972) and Stauffer and Wright (1984) have developed charts for this purpose. Stauffer and Wright used these charts and back-calculated shear strength parameters from a number of slides in embankments constructed of high-PI clays. These analyses were useful in establishing that the effective cohesion values were small for the embankments examined.

Duncan and Stark (1992) also back-calculated shear strength parameters using procedures similar to those described above. They back-calculated values for the Northolt Slip and found that the friction angles that were back-calculated exceed values determined in laboratory tests. They concluded that the procedure was not completely reliable, possibly because of the effects of progressive failure in the slope. Poor agreement may also have been caused by heterogeneity in the slope, which is common in natural slopes. Duncan and Stark also showed that the factor of safety changes only slightly with changes in the position of the slip surface, and thus the position of the slip surface is likely to be influenced to a significant degree by the normal variations in shear strength that occur in a slope.

Back-calculation of cohesion and friction angles by matching the computed critical slip surface with the observed location of the actual slip surface has met with only limited success and should be used cautiously. In many cases greater success is obtained by using other information, such as correlations between Atterberg limits and friction angles, to estimate one of the shear strength parameters and then to back-calculate the other. Several additional examples of back-analyses to determine slope conditions at the time of failure are presented in the next section.

Recapitulation

- Each combination of cohesion and friction angle that produces a factor of safety of 1 produces a unique location for the critical slip surface. Accordingly, the location of the slip surface can be used to calculate values for both cohesion (c, c') and friction angle (ϕ, ϕ').
- Use of the location of the slip surface to back-calculate both cohesion and friction has had mixed success and does not seem to work when there is significant progressive failure or distinct layering and inhomogeneities in the slope.

EXAMPLES OF BACK-ANALYSES OF FAILED SLOPES

The stability of any slope, including the results of any slope stability analysis, depends on numerous variables, including:

1. Unit weight of the soil
2. Loading conditions (i.e., whether the loading is undrained or drained)
3. Shear strength parameters, including whether the soil is anisotropic or the Mohr failure envelope is linear or nonlinear
4. Variability in the undrained shear strength or the shear strength parameters laterally and vertically
5. Seepage conditions and pore water pressures
6. Subsurface stratigraphy, including the presence of thin layers of soil with contrasting hydraulic or shear strength properties

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7. Shape of the slip surface
8. Method of analysis, including the assumptions made in the limit equilibrium procedure used

Some amount of uncertainty will exist in each of the foregoing variables, and the outcome of any analysis will reflect this uncertainty. If we seek to determine a shear strength parameter (c, c', ϕ or ϕ') by back-analysis, the value will reflect the uncertainty in all of the other variables that were used in the analysis. The degree of uncertainty in the shear strength parameter will be no less than the degree of uncertainty in all of the other variables that affect the stability analysis. In fact, the back-analysis should actually be conceived as a back-analysis to determine *all* of the variables that are applicable to the failure, rather than only shear strength. To reduce the uncertainty in this determination it is important to utilize all the information that is known or can be estimated by other means prior to performing the back-analysis. The back-analysis will then serve to establish reasonable values for all the variables.

Several examples are presented in this section to illustrate how available information is used in conjunction with back-analyses to establish a complete ''model'' of the slope at the time of failure. Some of these examples are of actual slopes or patterned after actual slopes and some are hypothetical.

Example 1: Embankment on Saturated Clay Foundation

The first example is of the cohesionless embankment (fill) slope resting on a deep deposit of saturated clay shown in Figure 12.8. The embankment has failed during construction, due to the underlying weak clay foundation. From knowledge of the fill material we can estimate that the friction angle for the embankment is 35° and the fill has a unit weight of 125 pcf. We can calculate the average undrained shear strength of the foundation by varying the assumed shear strength and calculating the factor of safety. From the results of such calculations it is determined that the average undrained shear strength is approximately 137 psf. Now, instead, suppose that we know from past experience with the soils in the area of the slope that the clay is slightly overconsolidated and that the undrained shear strength increases approximately linearly with depth at the rate of 10 psf per foot of depth. We can calculate a value for the undrained shear strength at the ground surface, assuming that the shear strength increases at the rate of about 10 psf per foot of depth below the surface. Doing so, we find that if the shear strength is approximately 78 psf at the ground surface and increases at the rate of 10 psf per foot of depth, the factor of safety will be 1. The two shear strength representations described above are plotted in Figure 12.9. Both

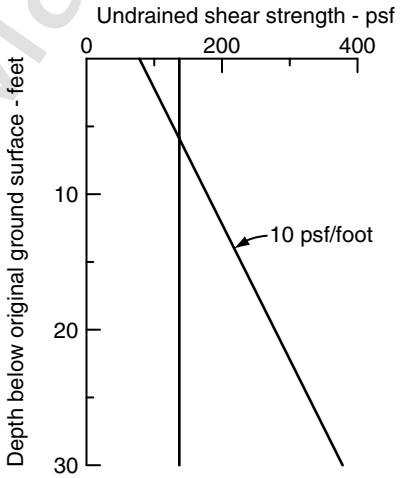


Figure 12.9 Undrained shear strength profiles from back-analysis of embankment on soft clay.

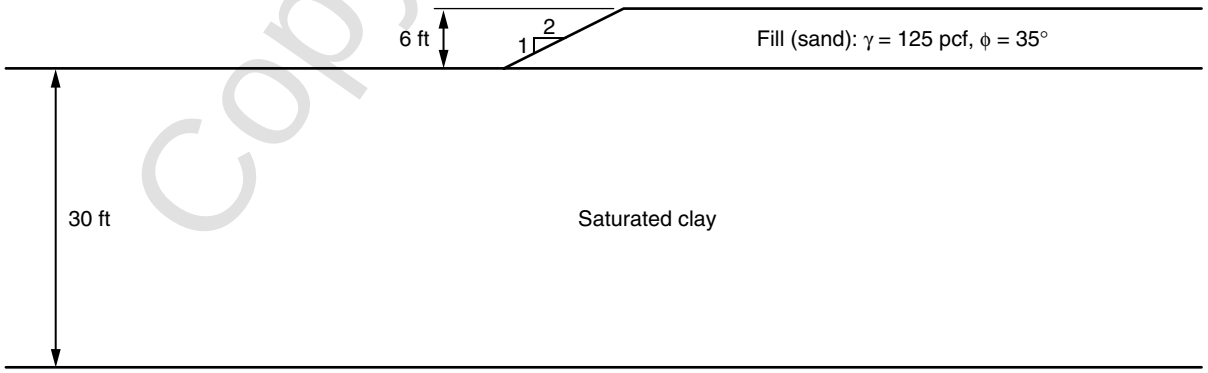


Figure 12.8 Embankment on soft clay foundation.

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of these representations give a factor of safety of 1. However, the locations of the corresponding critical slip surfaces are very different as shown in Figure 12.10. In addition, if we use the two shear strength representations to evaluate the effectiveness of reducing the slope height, we will reach different conclusions. Suppose, for example, that we want to increase the factor of safety to 1.5. Decreasing the slope height to 4 ft with the constant shear strength of 137 psf for the foundation is sufficient to achieve a factor of safety of 1.5. However, if the shear strength increases linearly with depth as represented by the second shear strength profile, the factor of safety is only increased to 1.3 by reducing the slope height to 4 ft. A factor of safety of 1.3 may very well be adequate for this embankment if the shear strength has been established from back-analysis, but if a factor of safety of 1.5 is necessary, the slope height must be reduced to something less than 4 ft.

For this example slope, knowledge of the shear strength of the embankment soil and how the shear strength varied with depth was used to establish a representation of shear strength. With the knowledge of the shear strength of the embankment, it was possible to calculate the shear strength of the foundation. Further, with knowledge of how the shear strength increased with depth, it was possible to establish a better representation of strength than was obtained when only an average (constant) shear strength was calculated. Without such information a greater amount of uncertainty would exist in the shear strengths determined by back-analysis.

Example 2: Natural Slope

The second example is of a natural slope located in the western United States. The soil profile consists of approximately 40 ft of weathered shale overlying unweathered shale (Figure 12.11). Substantial movement of the weathered shale was observed. The movement was believed to be taking place by slippage along the bottom of the weathered shale zone. Based on the movements that had already taken place, as well as experience with similar slopes in weathered shale, residual shear strengths were believed to be applicable. From the results of laboratory tests on the shale and correlations presented by Stark and Eid (1994), a residual friction angle (ϕ'_r) of 12° was estimated for the shale. In this case the shear strength parameters were believed to be relatively well known and the largest uncertainty was in the pore water pressure conditions in the slope. Therefore, a primary goal of the back-analysis was to estimate the seepage conditions in the slope that would be required to produce a factor of safety of 1. Because of the large lateral extent of the slope movements, infinite slope analysis procedures were used. Assuming a residual friction angle of 12° ($c' = 0$), it was found that a piezometric surface at a depth of approximately 12 ft below the ground surface would produce a factor of safety of 1. The actual water conditions in the slope varied widely over the large area of the slope, but groundwater observations in several borings were consistent with the back-calculated water level.

This slope was stabilized successfully and movement was halted by installation of a number of hori-

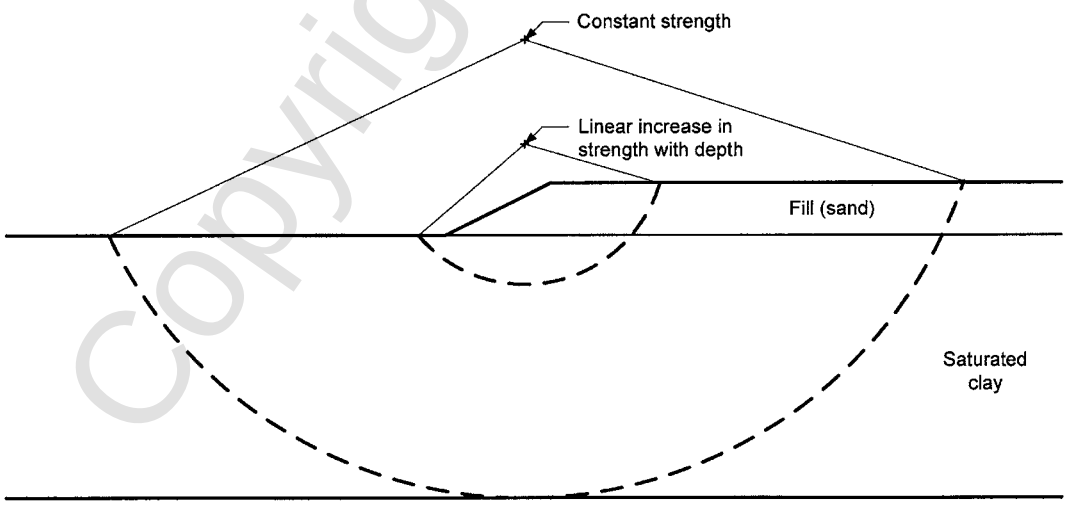


Figure 12.10 Critical circles from back-analysis of embankment on soft clay assuming constant undrained shear strength and linear increase in undrained shear strength with depth for foundation.

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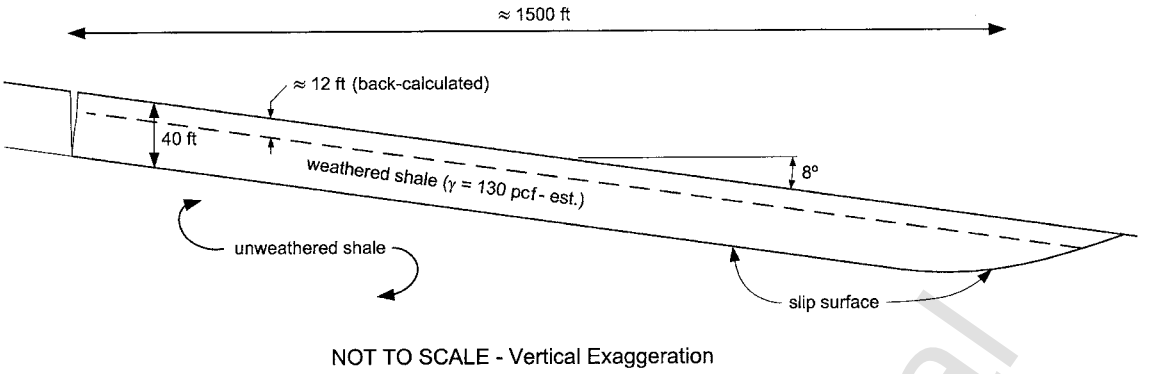


Figure 12.11 Natural slope in weathered shale.

zontal drains that lowered the water level. Lowering the water level 10 ft, from approximately 12 ft below the surface to approximately 22 ft below the surface, produced a factor of safety of 1.2. An increase from 1.0 to 1.2 was judged sufficient to stabilize the slope. In this case the back-analysis was used to confirm the residual shear strength values and determine the pore water pressure conditions at the time of failure. The resulting conditions then provided a basis for assessing the effectiveness of stabilizing the slope with horizontal drains.

Example 3: Earth Dam

The third example is of an earth dam. This example is patterned after the slide that occurred in the Victor Braunig Dam in San Antonio, Texas (Reuss and Schattenberg, 1972). Except for some minor adjustments in the geometry to simplify the problem, conditions are very similar to those of the actual dam. Information on the geometry and shear strength properties was taken from Reuss and Schattenberg. The cross section used for the present analyses is shown in Figure 12.12. The slide that occurred passed through the embankment and nearly horizontally along a clay layer near the top of the foundation. The slide occurred approximately five years after the dam was built. Due to layers and lenses of sand in the foundation of the dam, it was assumed that steady-state seepage was established in the foundation. Steady-state seepage was also assumed to have developed in the embankment, although the seepage conditions in the embankment did not have a major effect on the computed stability.

Analyses by Reuss and Schattenberg (1972) indicated that low residual shear strengths in the foundation contributed to the failure of the dam. Peak and residual shear strengths were measured for both the embankment soil and the clay in the foundation. Shear strength parameters from these tests are summarized in Table 12.2. A piezometric line representing the pore

Table 12.2 Shear Strength Parameters for Victor Braunig Dam Embankment and Foundation Clay Layer

	Peak strengths		Residual strengths	
Description	c' (psf)	ϕ' (deg)	c'_r (psf)	ϕ'_r (deg)
Embankment	400	22	200	22
Foundation clay	500	18	100	9

Source: Reuss and Schattenberg (1972).

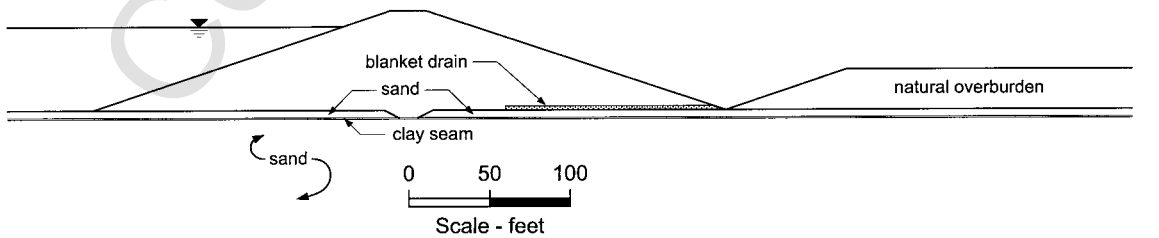


Figure 12.12 Cross section of dam on foundation with weak clay layer.

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water pressures in the embankment is shown in Figure 12.13. Pore water pressures in the foundation, particularly beneath the downstream half of the dam where the slide occurred, were assumed to be controlled by sand layers and lenses in the foundation. The sand layers and lenses were connected to the reservoir in the vicinity of the upstream toe of the dam. A simple linear piezometric surface that varied from the level of the reservoir at the upstream toe of the dam to the elevation of the ground surface at the downstream toe of the dam was assumed for the pore water pressures in the foundation (Figure 12.14). Factors of safety were calculated using both peak and residual shear strengths. With peak shear strengths the factor of safety was 1.78, while with residual shear strengths the factor of safety was 0.99. These calculations indicate that residual shear strengths probably developed in the foundation of the dam and contributed to the slide that occurred.

A significant source of uncertainty in the analyses described above was the pore water pressures in the foundation of the dam. Limited measurements of pore water pressures in the foundation were available, and these were in general agreement with the assumed piezometric levels chosen for the analyses. However, to determine if there might have been higher pore water pressures, and thus that possibly peak, rather than residual, shear strengths controlled the stability at the time of failure, additional analyses were performed using higher pore water pressures in the foundation of the dam. Two different piezometric lines chosen for these analyses are illustrated in Figure 12.15. Pore water pressures from these piezometric lines approach the overburden pressure in some areas of the downstream slope, and thus the piezometric lines are considered to represent an extreme condition. Stability calculations were performed using peak shear strengths and the two piezometric lines. Factors of safety for both piezometric lines were about 1.6 (range 1.56 to 1.58). Thus, it seems unlikely that peak shear strengths were applicable to this failure, and it was concluded that the earlier analysis with residual shear strengths was appropriate for the conditions at failure.

The slide in the Victor Braunig was stabilized successfully with a berm at the downstream toe. Analyses with a berm, residual shear strengths, and the piezometric lines shown in Figures 12.13 and 12.14 showed that the factor of safety was approximately 2.0 with the stabilizing berm in place.

Example 4: High-PI Clay Embankment

The failure of a highly plastic clay embankment was described earlier and illustrated in Figure 12.6. Back-analyses to calculate shear strength parameters that matched the location of the observed slip surface were shown to give $c' = 5$ psf, $\phi' = 19.5°$. After these analyses had been performed, consolidated–undrained

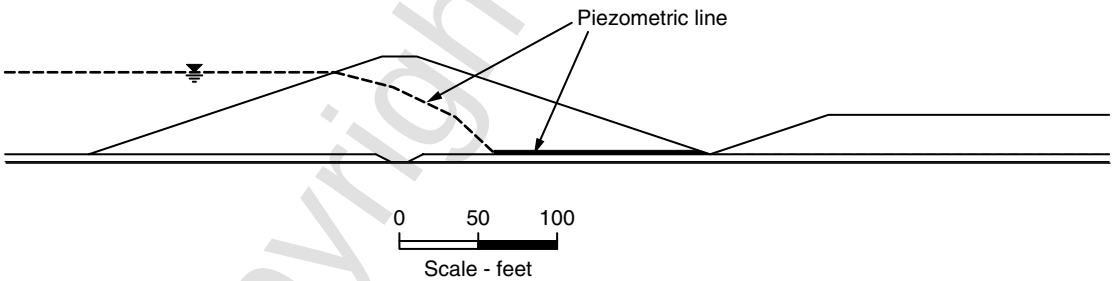


Figure 12.13 Piezometric line assumed for embankment of dam.

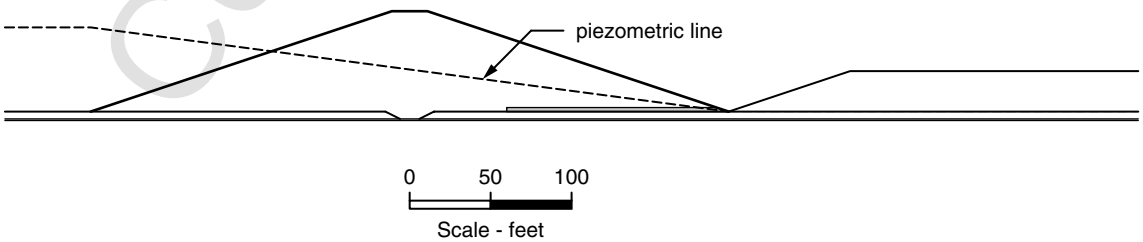


Figure 12.14 Piezometric line assumed for foundation of dam.

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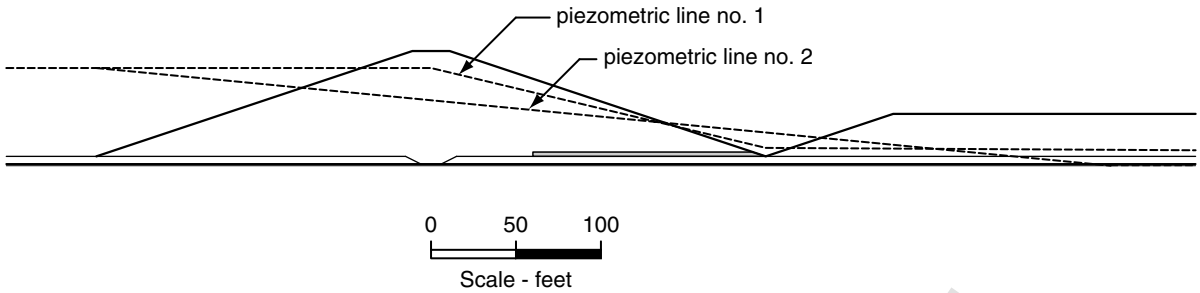


Figure 12.15 Alternative piezometric lines assumed for foundation of dam with peak shear strengths.

triaxial shear tests with pore water pressure measurements were performed to measure the effective stress Mohr failure envelope for the clay. The failure envelope determined from these tests is slightly curved and is shown in Figure 12.16. Also shown in this figure is the failure envelope that was back-calculated earlier from the slide geometry. The failure envelope that was back-calculated is substantially below the failure envelope that was measured in the laboratory tests. One possible explanation for the differences between the measured and back-calculated failure envelopes is that the pore water pressures were assumed to be zero for the back-calculated envelope. To determine if higher pore water pressures might explain the discrepancies, additional stability analyses were performed using the measured failure envelope and higher pore water pressures. For these analyses a piezometric line coincident with the slope face was assumed. This corresponds to uniform horizontal seepage in the entire slope and represents substantial pore water pressures in the slope. The factor of safety using this piezometric line and the measured shear strength envelope shown in Figure 12.16 was approximately 2.0. Thus, it seems highly unlikely that the discrepancy between the measured and back-calculated shear strengths was due to the assumption of zero pore water pressures used in the back-analyses.

Because of the apparent discrepancies between the back-analyses and the laboratory measurements of shear strength, additional laboratory tests were undertaken. These additional tests on the fill material from the embankment showed that significant softening of the soil occurred when it was subjected to repeated wetting and drying that ultimately led to a lower, ''fully-softened'' shear strength. The Mohr failure envelope for the fully-softened clay is shown in Figure 12.17. The failure envelopes for peak strength as well as the failure envelope back-calculated earlier are also shown in this figure. The fully softened strength can

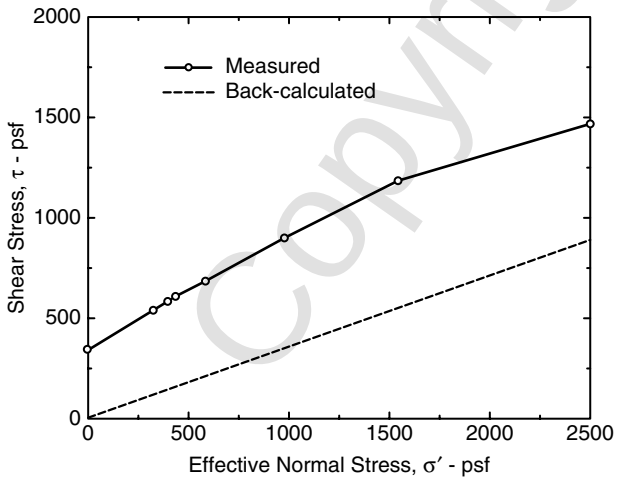


Figure 12.16 Measured failure envelope and failure envelope determined by back-analysis using the critical slip surface location.

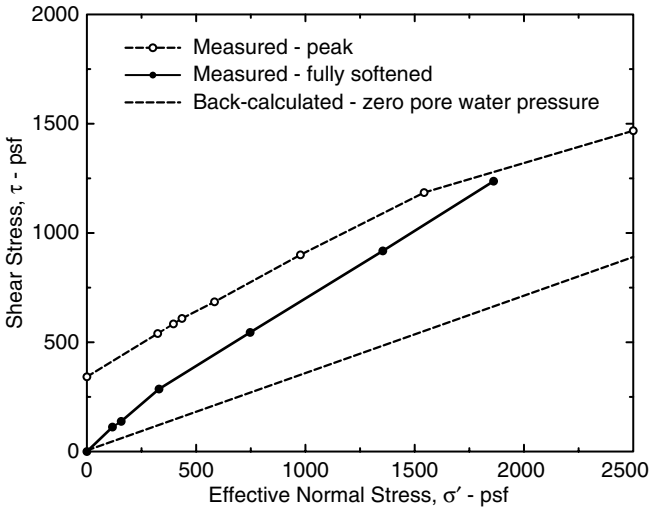


Figure 12.17 Measured fully softened and peak failure envelopes and failure envelope determined by back-analysis using the critical slip surface location.

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be seen to be significantly less than the peak strength; however, the fully softened strength is still much higher than the strength that was back-calculated. One possible explanation could again be that the pore water pressures were assumed to be zero in the back-analysis.

To determine if pore water pressures could possibly explain the discrepancies between the fully-softened and back-calculated shear strength envelopes, additional analyses were preformed using the fully-softened strength envelope and assuming various pore water pressure conditions. Pore water pressures represented by a piezometric line coincident with the slope face were found to produce a factor of safety of approximately 1 with the fully softened strength. Based on these analyses, it thus appears that the fully softened shear strengths are applicable and that relatively high pore water pressures were developed in the embankment. The slide in the embankment occurred following a period of wet weather and significant rainfall. It is likely that high pore water pressures developed in the slope at least temporarily. Thus, the back-analyses combined with laboratory testing to measure the fully softened shear strength of the soil were used to establish the probable conditions in the slope when failure occurred. The earlier back-analyses in which the pore water pressures were assumed to be zero and the shear strength parameters were calculated to match the depth of the assumed slip surface were useful in establishing that the shear strengths were relatively low, less than the measured peak shear strengths. However, the initial back-analyses did not fully explain the conditions at failure; only after further laboratory testing and use of the results from the laboratory tests in back-analyses to determine the pore water pressures were the conditions in the slope better understood.

Example 5: Kettleman Hills Landfill Failure

The final example is the Kettleman Hills landfill failure. Mitchell et al. (1990), Seed et al. (1990), Byrne et al. (1992), and Stark and Poeppel (1994) have discussed this failure extensively. One issue that has emerged from the studies by various investigators is whether peak or residual shear strengths were developed in the liner system along the base of the waste fill. Gilbert et al. (1996b) reanalyzed this failure using probabilistic methods. Rather than calculate a single value for the factor of safety based on assumed conditions, they considered the probability of failure. Their analyses accounted for the uncertainties in shear strength due to variability in measured peak and residual shear strengths, as well as the uncertainty associated with the method of analysis, including the effect of interslice force assumptions and potential three-dimensional effects. For their analyses, Gilbert et al. expressed the strength relative to peak and residual values, by a *mobilized strength factor,* R_s, defined as

$$R_s = \frac{s - s_{\text{av}.r}}{s_{\text{av}.p} - s_{\text{av}.r}} \tag{12.5}$$

where s is the shear strength, $s_{\text{av}.r}$ the mean residual shear strength, and $s_{\text{av}.p}$ the mean peak shear strength. Envelopes for both the peak and residual shear strengths were assumed to possess random variability, and a normal distribution was assumed; values from the mean failure envelopes were used to compute R_s. If the residual shear strength is developed, R_s is zero, and if the peak strength is developed, the value of R_s is 1.

Gilbert et al. (1996b) performed analyses for the cross section shown in Figure 12.18 and calculated the probability of failure as a function of the value for the factor R_s. Their results are summarized in Figure 12.19. The most probable value of R_s is 0.44, indicating that the shear strength developed at failure was approximately halfway between the peak ($R_s = 1$) and residual ($R_s = 0$) values. Previous studies by Byrne et al. (1992) and Stark and Poeppel (1994) had concluded that peak shear strengths were developed along the base of the landfill, while another study by Gilbert et al. (1996a) concluded that residual shear strengths were developed. The subsequent analyses by Gilbert et al. (1996b) indicate that the probabilities of peak and residual shear strengths being developed are approximately equal. However, the analyses also indicate that probably neither peak nor residual shear strengths, but rather some intermediate values of shear strengths, were developed. The analyses indicated that a progressive failure probably took place, and this is supported by subsequent finite element analyses by Filz et al. (2001).

Back-analyses of the Kettleman Hills landfill using probabilistic methods were helpful in understanding the failure that occurred and what shear strengths were developed at the time of failure. However, for analysis and design of remedial measures in this case, it is probably more appropriate to use residual shear strengths rather than the higher strengths that were developed at the onset of failure, because the slope experienced relatively large deformations.

Summary

In each of the examples presented above, some information about the shear strength parameters or pore water pressure conditions was available to guide the back-analyses. This information, along with the knowledge that the factor of safety was 1, was used to arrive

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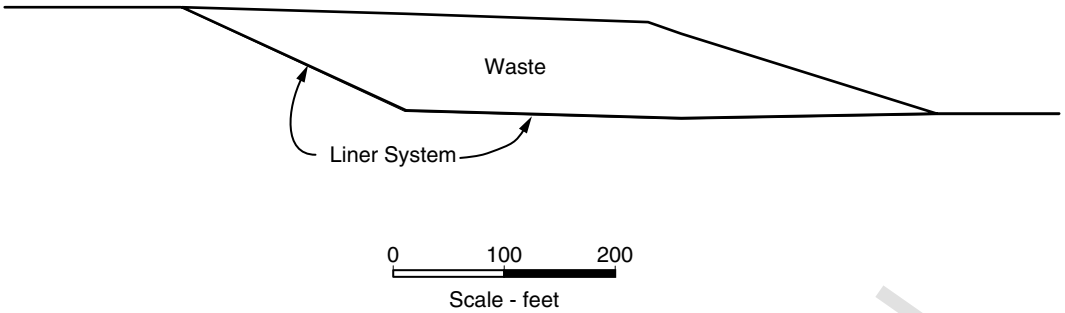


Figure 12.18 Prefailure cross section of Kettleman Hills landfill.

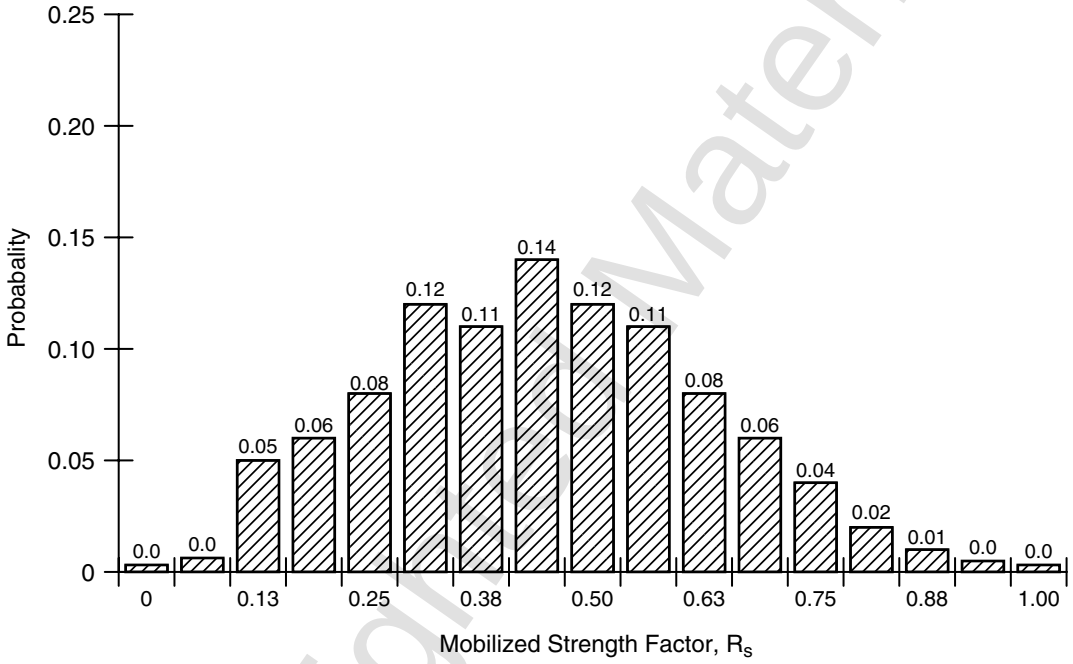


Figure 12.19 Probabilities of failure for various relative amounts of mobilized peak and residual shear strengths in Kettleman Hills landfill liner.

at a complete set of conditions that were believed to be representative of those in the slope at the time of failure. In three of the five examples, the pore water pressures at failure were uncertain, and some information was available about the shear strength parameters. Back-analyses were used to establish the pore water pressures as well as to confirm the values of the shear strength parameters.

Duncan (1999) presented results of back-analyses for three case histories where procedures similar to those described above were used to establish the conditions in the slope at the time of failure. These three examples are summarized in Table 12.3 along with the five examples presented in the preceding section. For each example the important conditions that were established by the back-analyses are indicated. In each of these cases there was at least some uncertainty in several variables, and the back-analyses served to establish a reasonable set of conditions at the time of failure. Also, by performing the analyses with the same limit equilibrium procedures used subsequently to analyze the remedial measures, the validity of the computational procedure was established along with the soil and slope properties. This validation gave increased confidence in the analyses that were performed to evaluate remedial measures.

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Table 12.3 Summary of Examples of Back-Analyses and Results

Example	Conditions defined by back-analysis
Example 1: embankment on saturated clay foundation	Back-analyses established the undrained shear strength at the ground surface using estimated rates of strength increase with depth.
Example 2: natural slope	Back-analyses confirmed residual shear strengths and established a piezometric level in agreement with groundwater observations.
Example 3: earth dam	Back-analyses showed that residual shear strengths were developed and established piezometric levels used later for evaluation of remedial measures.
Example 4: high-PI clay embankment	Initial back-analyses showed that negligible cohesion intercept existed. Further analyses supported by laboratory data showed that shear strengths were reduced from peak to fully softened and that relatively high pore water pressures must have existed at the time of failure.
Example 5: Kettleman Hills landfill failure	Probabilistic analyses were used to establish that the shear strength developed at failure was somewhere between peak and residual values and suggested that progressive failure occurred.
San Luis Dam (Duncan, 1999)	Back-analyses established that residual shear strengths were developed in highly plastic clay slopewash.
Olmsted landslide (Duncan, 1999; Filz et al., 1992)	Back-analyses established anisotropic shear strength values for McNairy I formation and piezometric levels in slope.
La Esperanza Dam (Duncan, 1999)	Back-analyses established piezometric levels and confirmed estimated shear strengths of brecciated shale.

PRACTICAL PROBLEMS AND LIMITATION OF BACK-ANALYSES

Back-analyses can provide a useful insight into the conditions in a slope at the time of failure; however, several limitations and factors can complicate such analyses. These are discussed below.

Progressive Failure

A fundamental assumption in all limit equilibrium slope stability analyses is that the shear strength is mobilized simultaneously along the entire slip surface. If a single set of shear strength parameters (i.e., a single value for c and ϕ) is assumed, while in reality the values vary as a result of progressive failure, the back-calculated values represent only an ''average'' of the shear strength parameters that were mobilized on the failure surface; the average may not represent the actual shear strength parameters at any point on the failure surface. This is very likely to be the case where progressive failure occurs. As Duncan and Stark (1992) showed for the analysis of the Northolt Slip, the shear strength parameters back-calculated from the slide geometry did not agree with either peak or residual shear strength values. Gilbert et al. (1996b) showed similar results for the Kettleman Hills landfill failure. In the case of Kettleman Hills, probabilistic methods were useful in understanding the shear strengths that were developed, as were the finite element analyses presented by Filz at al. (2001).

If progressive failure occurs, the back-calculated shear strengths are likely to be inappropriate for redesign. For most slopes where progressive failure occurs, the movements are likely to be large enough that once the slide movement has ceased, the strengths are reduced to residual values, and residual values should be used for redesign even though higher values may be determined by back-analysis. The use of average values calculated from the initial slide geometry may be unconservative.

Decreasing Strengths with Time

In most cases back-analyses are performed for either short-term conditions with undrained shear strengths or for long-term conditions with effective stress shear strength parameters and steady-state seepage or groundwater levels. The corresponding back-analyses assuming the appropriate one of these conditions, however, may not result in the most critical value of shear strength. For example, consider an excavated slope in clay that fails during construction. Ordinarily, undrained conditions would be assumed and used to back-calculate shear strengths for such a slope. The undrained shear strength calculated from such analyses should reflect the shear strength when the failure oc-

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curred. If the slope was excavated very recently, or if it fails during construction, the soil is probably undergoing swelling (expansion) and the shear strength and stability will continue to decrease after failure occurs. For redesign, a strength significantly lower than the one determined by back-analysis may be appropriate.

Even when slopes fail a number of years after construction, the strength that is calculated only represents the strength at the time of failure, and the strength may still be decreasing further. For back-analysis of slopes that fail a number of years after they are built, the pore water pressures are often assumed based on an estimated groundwater level, and if seepage occurs, the seepage is assumed to have reached a steady-state condition. This is probably the case with early investigations of a number of slopes in London Clay. Skempton (1964) first suggested that many slopes that failed in London Clay failed at different times after construction because progressive failure was occurring and the strength (effective stress shear strength parameters) was decreasing with time. Although Skempton did not clearly indicate what pore water pressure conditions were assumed for his analyses, it appears that steady-state seepage was assumed and that groundwater levels were assumed to have reached steady-state equilibrium levels. Vaughan and Walbancke (1973) later measured pore water pressures in slopes in London Clay and showed that the pore water pressures increased gradually over many years. This led to the realization that a significant portion of the time-related delay in failure may have been due to changes in pore water pressure; progressive failure over time may actually have played only a small role in the failures.

Complex Shear Strength Patterns

Back-calculation of shear strength in most cases consists of back-calculating one quantity (c or ϕ) or at most two quantities (c and ϕ) to represent the shear strength of the soil. In reality the shear strength is generally more complex and knowledge of the shear strength is helpful in establishing what should be back-calculated. For example, for the Example 1 slope shown in the preceding section, the undrained shear strength was known to increase with depth, and when this was taken into account, very different results were obtained compared to what was found when the shear strength was assumed to be constant.

In addition to the shear strength varying with depth, there are other forms of shear strength variations that may affect and complicate the back-calculation of shear strength. One such case is where the shear strength varies with the orientation of the failure plane (i.e., where the shear strength is anisotropic); another case is where the shear strength varies nonlinearly with the normal stress (i.e., where the Mohr failure envelope is curved). In both these cases, back-calculation of a single c or ϕ value may lead to significant errors that vary with the location of the slip surface. If the slip surface that was used for the back-analysis is very different in its orientation or depth from the critical slip surface for the redesigned slope, the back-calculated shear strengths may not be applicable and may not reflect the proper shear strength for other than the original slip surface.

In back-calculating shear strengths it is important to have the proper model for the shear strength. Laboratory data or estimates of shear strength based on correlations between shear strength and index properties can be useful in estimating shear strength parameters. It is also essential to know whether the shear strengths should be represented by undrained shear strength parameters and total stresses or by drained shear strengths and effective stresses. Similarly, it is important to know if:

1. The soil is likely to be anisotropic and anisotropy will play a significant role in the location of the failure surface.
2. The shear strength envelope is curved such that c and ϕ are stress dependent.
3. The undrained shear strength ($s_u = c$, $\phi = 0$) is constant or varies significantly with depth.

As noted previously, it is also important to judge whether the appropriate shear strength to be calculated is the undrained shear strength or the drained shear strength, and if the shear strength may decrease after failure.

Recapitulation

- Back-calculated shear strengths must be used cautiously if progressive failure has occurred.
- Laboratory data and experience can provide useful information to guide the back-calculation of shear strengths. Even when laboratory data are not available, it is possible to make reasonable estimates of the friction angle, ϕ', based on index properties.
- If the shear strengths are decreasing significantly at the time of failure due either to changes in pore water pressure or softening of the soil structure, the back-calculated shear strengths may not be appropriate for use in designing remedial measures.
- It is important to assume the appropriate model to back-calculate the shear strength parameters. Curved Mohr failure envelopes and anisotropy may influence the validity of back-calculated shear strengths.

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OTHER UNCERTAINTIES

Slope stability analyses can involve numerous uncertainties, and some of the uncertainties can be difficult to quantify. One of the benefits of back-analyses is that many of the same errors exist for both the back-analysis and the redesign. Thus, there are compensating effects and the net result of the errors is diminished or removed entirely. This needs to be kept in mind when the results from back-analysis are compared with data obtained by other means. For example, results of laboratory tests may not compare favorably with results of back-analysis if there are significant three-dimensional effects that were not considered in the slope stability analyses. However, if the slope is to be redesigned using two-dimensional analyses and, again, three-dimensional effects are neglected, the back-calculated values may be the more appropriate values to use. Caution must also be exercised, however, because neglect of three-dimensional effects will cause back-calculated shear strengths to be too high, and if comparable three-dimensional effects do not exist for the slope redesign, the results may be on the unsafe side.

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